Hankel-type Model Reduction Based on Frequency Response Matching

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Abstract—In this paper, a stability preserving model reduction algorithm for single-input single-output linear time invariant systems is presented. It performs an interpolation from the frequency domain data using semidefinite programming methods. Calculating the frequency response of the model can be done efficiently even for large scale models, making this approach applicable to those. The relaxation used to obtain a semidefinite program is similar to one used in Hankel model reduction. Therefore the accuracy of approximation is also similar to Hankel model reduction one. The approach can be easily extended to frequency-weighted and parameter-dependent model reduction problems.

I. INTRODUCTION

Model order reduction has received considerable attention in the past and there exists a number of established techniques to obtain low-order approximations. Most of the existing methods fall into two categories: SVD-based (singular value decomposition) and Krylov-based methods. The SVD-based methods, which incorporate balanced truncation and Hankel model reduction, are mostly used for approximation of low and medium scale models. Balanced truncation ([11]) proposes a simple, yet a very powerful algorithm with a stability guarantee for the reduced model and the approximation error bounds. Hankel model reduction ([2]) is much more complicated than the latter, but on the other hand numerically more robust and solves a suboptimal problem with the error bounds tighter than the balanced truncation ones. Both methods rely on the solution of Lyapunov equations to calculate the approximation, which makes them numerically heavy. The Krylov-based methods (see, [3], [4]) rely on moment matching techniques and therefore provide much cheaper solutions, however, there exist issues with stability of approximations. These issues were solved with Krylov/SVD-based methods (see, [5]), which enforce stability on approximations for an extra computational cost.

All the described methods calculate an approximation from state-space representations of the full models. Instead, one can use the frequency domain data i.e., the frequency response samples, to reduce the full model. Calculating the frequency response for particular applications (e.g. modeling of electro-mechanical structures) can be even cheaper, than inverting the state-space matrix A, as shown in [6], [7], [8]. The approximation from the frequency data is related to the celebrated Nevanlinna-Pick interpolation problem, an extension of which to Hardy spaces can be found in [9] with recent progress in [10]. In [11], another approach was developed to obtain an approximation. It is based on convex optimization and exploits a Hankel-type relaxation to obtain a semidefinite formulation. In this paper, a generalization of [11] is proposed, which has smaller lower and upper error bounds than the predecessor. The proposed method, as well as the one in [11], can be regarded as an extension of Hankel model reduction to large scale model approximation. The semidefinite optimization approach is valuable due to simplicity of possible extensions, e.g. frequency-weighted and parameter-dependent model reduction.

The paper is organized as follows. In Sec. II, model reduction is re-formulated as a bilinear optimization problem. These results are strictly theoretical, although there exist methods to solve such problems as [12]. In Sec. III the proposed Hankel-type relaxation is described. Sec. V implementation issues are discussed and finally examples are presented in Sec. VI.

Notation

\(\mathcal{H}_\infty\) and \(\mathcal{H}_\sim\) stand for the space of discrete-time stable and antistable transfer functions correspondingly. Operation \(\sim\) denotes the adjoint in \(\mathcal{H}_\infty\) space i.e., \(G^\sim(z) = G^T(1/z)\). \(G(\omega)\) stands for the frequency response of \(G(e^{j\omega})\) to \(\omega \in [0, \pi]\). The infinity norm is computed as \(\|G\|_\infty = \sup_{\omega \in [0, \pi]} |G(\omega)|\), where \(G(\omega)\) is a scalar-valued function. The unit disc and the unit circle in the complex plane are denoted as \(\mathbb{D} = \{z| |z| < 1\}\), and \(\partial \mathbb{D} = \mathbb{D} = \mathbb{D}/\mathbb{D}\) respectively.

By optimal Hankel model reduction (HMR) throughout this paper is meant the following algorithm. First, compute the Hankel approximation of \(G\), denoted as \(G_h = p_h/q_h\), where \(p_h\) and \(q_h\) are polynomials in \(z^{-1}\). Then subject the numerator \(p_h\) to a further optimization:

\[
\gamma_h = \min_p \|G - p/q_h\|_\infty
\]

where \(\gamma_h\) is the optimal approximation level and the reduced model is \(p_0/q_h\), where \(p_0\) is a solution to the latter problem. The original Hankel reduction will be referred as “pure” HMR.

The quasi-convex optimization (QCO) approach from [11] forms the following algorithm. Solve the relaxed problem,

\[
\gamma_{qco} = \min_{\gamma > 0, a,b} \gamma
\]

subject to \(|G(\omega)a(\omega) - b(\omega)| \leq \gamma a(\omega)\quad \forall \omega \in [0, \pi]\)

where \(b = \sum_{i=k}^{k} b_i z^i\), \(a = 1 + \sum_{i=1}^{k} a_i(z^i + z^{-i})\) and \(\gamma_{qco}\) is a sub-optimal approximation level. Then given \(a\), solve a
spectral factorization problem \( a = qq^* \), where \( q \) has only stable zeros and poles. Finally using the obtained \( q \) solve,

\[
\min_p \|G(\omega) - p(\omega)/q(\omega)\|_\infty
\]

to calculate the numerator of the reduced model.

II. Preliminaries

The main focus of this paper is reduction of discrete time models in the frequency domain. The problem for scalar valued transfer functions can be formulated as minimization:

\[
\min_{p,q} \|G - p/q\|_\infty
\]

where \( p = \sum_{i=0}^{k} p_i e^{-i\omega} \), \( q = 1 + \sum_{i=1}^{k} q_i e^{-i\omega} \) and \( q \) is a minimum phase transfer function. Minimizing the \( H_\infty \) norm can be rewritten as a minimization of the approximation level \( \gamma \) with a norm constraint enforced for all the frequencies \( \omega \):

\[
\gamma_{m_m} = \min \gamma
\]

subject to \( |G(\omega)q(\omega) - p(\omega)| \leq \gamma|q(\omega)| \quad \forall \omega \in [0, \pi] \)

where \( q \) is minimum phase

The minimum phase condition here is equivalent to stability of the reduced model \( p/q \), which has the order less or equal to \( k \). This problem is known to be non-convex and some successful convex relaxation techniques have been proposed (e.g., [11]).

Another way to obtain a convex problem is using a restriction on decision variables as in [13]. Instead of the minimum phase constraint on \( q \), consider a positive real one i.e., \( \text{Re}(q) \) is positive for all \( \omega \). This constraint is convex and can be expressed as a semidefinite one, unlike the minimum-phase condition. Now the only obstacle in obtaining a convex program is the right-hand side of the norm constraint \( |G(\omega)q(\omega) - p(\omega)| \leq \gamma|q(\omega)| \). If \( q \) would be positive then the minimization would correspond to a second order cone program, which is convex. In [13] it was proposed to substitute \( |q| \) with \( \text{Re}(q) \), since it has been already parameterized as a positive polynomial. Finally the norm constraint is reformulated as:

\[
|G(\omega)q(\omega) - p(\omega)| \leq \gamma \text{Re}(q(\omega))
\]

In [13] it was shown that the positive real constraint on the denominator \( \text{Re}(q) \) may be very restrictive. In fact, as long as the full system has poles near the unit circle (which is often the case) the approximation will be too conservative.

To address this limitation introduce a new frequency dependent variable \( \varphi \) into the minimization as:

\[
\gamma_{bt} = \min_{\gamma > 0, p,q, \varphi} \gamma \quad \text{subject to}
\]

\[
|G(\omega)\varphi(\omega) - p(\omega)\varphi(\omega)| < \gamma \text{Re}(q(\omega)\varphi(\omega)) \quad \forall \omega
\]

where \( q \) is minimum phase

Surprisingly this bilinear formulation is equivalent to the general model reduction problem (4), so\( \gamma_{bt} \) is an optimal solution. Furthermore, instead of the condition \( q \) is a stable polynomial, a more convenient bilinear condition \( q/\varphi^* \) is a strictly positive real (SPR) transfer function is obtained. This program can also be implemented using nonlinear optimization, for example, a branch-and-bound method implemented in [12].

Solving a bilinear program, however, is not the best solution to the model reduction problem due to nonlinearity and nonconvexity. Numerical simulations with the software package [12] show that it can be used for the problems with a low number of decision variables. In fact, in most of the examples presented in Sec.VI, the solution did not converge to optimality in reasonable time. Therefore there is a need for a semidefinite relaxation.

III. Optimization-based Hankel-Type Model Reduction (OHMR)

Consider the program (5) and a straightforward convex relaxation i.e., introducing the new variables \( a := qp \) and \( b := pp \). This yields an algorithm:

\[
\gamma_{ohmr} = \min_{\gamma > 0, a,b} \gamma
\]

subject to \( |G(\omega)a(\omega) - b(\omega)| \leq \gamma \text{Re}(a(\omega)) \quad \forall \omega \)

where \( a = \sum_{i=k}^{k} a_i e^{i\omega} \) and \( b = \sum_{i=-k}^{k} b_i e^{i\omega} \). This problem is convex in \( a_i \) and \( b_i \), as a second order cone program. The non-convex condition \( q \) is minimum-phase, which corresponds to \( a \) has exactly \( k \) stable zeros, is impossible to parameterize in a convex manner in \( a \) and \( b \). Remarkably this constraint becomes redundant using the following statement,

**Lemma 3.1:** Consider a function \( a = \sum_{i=-k}^{k} a_i z^i \) and a closed encircling the origin contour \( \partial \mathbb{D} \). If \( \text{Re}(a(\partial \mathbb{D})) > 0 \) then the polynomial \( a \) has exactly \( k \) zeros in \( \mathbb{D} \) and no zeros on \( \partial \mathbb{D} \).

*Proof:* The function \( a(z) \) does not have zeros or poles on the contour (since \( \text{Re}(a(\partial \mathbb{D})) > 0 \)) and it is analytic in \( \mathbb{D} \), except for a set of isolated points. Thus, by Cauchy’s argument principle, \( N_s - N_p = N_o \), where \( N_s \) is the number of zeros in \( \mathbb{D} \), \( N_p \) is the number of poles in \( \mathbb{D} \), and \( N_o \) is a
winding number of \( a(\partial \mathbb{D}) \) (number of times \( a(\partial \mathbb{D}) \) encircles the origin). Since \( \text{Re}(a(\partial \mathbb{D})) > 0 \) for all the frequencies \( \omega \), the curve \( a(\partial \mathbb{D}) \) lies only in the right half plane and thus \( N_\omega = 0 \). Since \( N_p = k \) the result follows.

Note that instead of \( \mathbb{D} \) any set satisfying the Cauchy’s argument principle may be used. The statement may also be applied to the problem (5). Similarly, if \( \text{Re}(q \varphi) > 0 \) for all frequencies, exactly \( k \) zeros will lie inside the unit circle. However, there is no guarantee that it is \( q \), that will have all the zeros inside the unit circle.

The denominator \( q \) is obtained through decoupling the stable modes of \( a \) by solving the equation

\[
a = q \varphi
\]

where \( \varphi \) has all the unstable zeroes of \( a \), also \( q \) is of order \( k \) as stated in Lemma 3.1. Now it is clear that the transfer function \( b/a \) has stable and antistable parts both of order \( k \). Since the optimization is also performed over both, this approach may be regarded as a suboptimal model reduction in Hankel norm, i.e. suboptimal Hankel approximation. Finally the stable approximation of \( G \) is obtained by another minimization

\[
\min_p \| G - p/q \|_\infty
\]

The described algorithm still has an infinite number of constraints, since the conditions are enforced for all the frequencies \( \omega \in [0, \pi] \). However, usually, the function \( G \) is “good” (rational in \( z \) and without poles on \( \partial \mathbb{D} \)), therefore it is sufficient to impose the constraints in the finite number of points \( \omega_i \). A detailed description of this approach may be found in Sec. V.

Note, if one fixes \( a \) to be a symmetric (pseudo-) polynomial, then the proposed approach reduces to the QCO method from [11]. Therefore the OHMR method has smaller lower and upper error bounds than the QCO one.

### IV. Positive Real Denominator Method Using a Different Basis

As was mentioned earlier there is an ad-hoc way of choosing \( \varphi \), which contains the adjoint of the resonant poles \( \zeta_i \) of the full system \( G \). Note, that the reduced model does not depend on \( \varphi \), therefore the chosen poles will not be a part of the reduced model. This procedure appears more numerically robust than just adding the poles as a weight.

Addition of \( \varphi \) can be generalized by changing the basis of the polynomials \( p \) and \( q \). Multiplying with \( \varphi \) is equivalent to the change of basis from \( z^{-1} \) to \( F_i(z) = z^{-i} \varphi \). One can also normalize the basis as \( F_i(z) = z^i |\varphi|/\varphi^* \). This way \( \varphi \) is incorporated in procedure and the basis is still orthonormal. The polynomials are parameterized as,

\[
p = p_0 + p_1 F_1(e^{2\pi i}) + \cdots + p_k F_k(e^{2\pi i})
\]

\[
q = q_0 + q_1 F_1(e^{2\pi i}) + \cdots + q_k F_k(e^{2\pi i})
\]

Another possibility of choosing the basis is using the poles \( \zeta_i \) directly. The basis itself may be a partial fraction (i.e. \( F_i = \frac{1}{z - \zeta_i} \)) or a generalized orthonormal one. The latter approach

may provide a better numerical conditioning as stated in [14]. Generalized orthonormal basis functions are defined as follows. If \( \text{Im}(\zeta_i) = 0 \), then \( F_i(z) = \sqrt{1 - |\zeta_i|^2}/z - \zeta_i \). If \( \text{Im}(\zeta_i) \neq 0 \), then

\[
F_i(z) = \frac{\sqrt{1 - |\zeta_i|^2}}{\sqrt{1 + |\zeta_i|^2}} \left( 1 + |\zeta_i|^2 \right) z - 2 \text{Re}(\zeta_i) \psi_{i-1}(z) - 2 \text{Re}(\zeta_i) z + |\zeta_i|^2 \psi_{i-1}(z)
\]

\[
F_{i+1}(z) = \frac{\sqrt{1 - |\zeta_i|^2}}{\sqrt{1 + |\zeta_i|^2}} \left( 1 + |\zeta_i|^2 \right) z^2 - 2 \text{Re}(\zeta_i) z + |\zeta_i|^2 \psi_{i-1}(z)
\]

Functions \( \psi_i(z) = \prod_{j=1}^i \frac{1 - z \zeta_j}{z - \zeta_j} \) are the Blaschke products and thus inner (all-pass) functions (\( |\psi_i| = 1 \)). The number \( a_j \) is a permutation of \( 1, \ldots, k \). The most numerical robust permutation may be chosen as in [14].

The proposed approach can also be used for the Hankel-type reduction methods. However, using it together with the PRD method provides more benefits. Recall, that the PRD was restrictive due to the positive real constraint on the denominator of the model. By switching the basis this problem is solved. Moreover, now it is possible to enforce the constraints directly on the transfer function coefficients of the reduced model, which is generally hard to do with the Hankel-type reduction. Also an extension to parameterized model reduction can be made as in [13], that is able to obtain explicit parameter-dependent transfer functions unlike the QCO or OHMR methods. The approach described in this section will be referred as the PRD method.

### V. Implementation

In order to obtain a tractable optimization problem the constraints are imposed only on a finite frequency grid \( \{ \omega_i \}_{i=1}^N \subset [0, \pi] \). The number of points should be \( O(k^2) \), where \( k \) is the order of the approximation, to avoid over-fit. This approach may create unstable approximations, therefore the positivity constraint \( \text{Re}(a) > 0 \), which guarantees stability of the reduced models, is enforced for all the frequencies. It may be done efficiently using the KYP lemma, e.g using the formulation from [15].

The algorithm is as follows: solve,

\[
\begin{aligned}
\min_{a,b,\gamma} \gamma & \quad \text{subject to} \\
|G(\omega_i)a(\omega_i) - b(\omega_i)| & \leq \gamma \text{Re}(a(\omega_i)) \quad i = 1, \ldots, N \\
\text{Re}(a) & > 0 \quad \forall \ \omega \in [0, \pi]
\end{aligned}
\]

Given \( a \), perform a stable-antistable factorization,

\[
a = q \varphi
\]

where \( \varphi \) has only unstable zeros and poles and \( q \) only stable ones. Finally using the obtained \( q \) solve,

\[
\min_p \max_{i=1,\ldots,N} |G(\omega_i) - p(\omega_i)/q(\omega_i)|
\]

Surely, the norm constraint (10) could also be enforced for all the frequencies using the same techniques as in the case of (11). However, then the size of the norm constraint
LMI would be four times the order of the full model $G$. Moreover, solving the norm constraint LMI would be even more expensive than solving Lyapunov equations of the corresponding sizes. Recall, that HMR requires to solve the Lyapunov equations of dimensions twice as the order of the full model. Thus there would be no advantages with respect to HMR. Therefore this approach is not considered.

A. Positive Real Denominator (PRD) Method

Consider the bilinear formulation of model reduction (5). Assume that $\varphi$ is fixed and predefined. Parameterize the polynomials $p$ and $q$ in a new basis as:

$$p = p_0 + p_1 F_1(e^{j\omega}) + \cdots + p_k F_k(e^{j\omega})$$

$$q = q_0 + q_1 F_1(e^{j\omega}) + \cdots + q_k F_k(e^{j\omega})$$

where basis functions $F_i$ may be chosen as $F_i = z^i/\varphi^\sim$ or as generalized orthonormal ones, which provide better numerical conditioning (see [14], for details). Note that the choice $F_i = z^{-i}$ yields the method from [13]. It should be noted that a predefined basis $F_i$ does not affect the efficiency of the algorithm, since the original formulations do not require any specific basis. Given the described above parameterization, the minimization is set up as follows,

$$\min_{p,q,\gamma} \gamma \quad \text{subject to} \quad \begin{align*}
|G(\omega)q(\omega_i) - p(\omega_i)| &\leq \gamma \Re(q(\omega_i)) & i = 1, \ldots, N \\
\Re(q) &> 0 & \forall \omega \in [0, \pi] \quad \text{and}
\end{align*}$$

(14)

(15)

VI. EXAMPLES

These examples are set to estimate the actual relaxation gap in the Hankel-type approximation and a possible improvement by PRD with the basis approach to any reduction procedure.

Some of the models in the considered examples are continuous-time systems. Since the proposed approaches (as well as the QCO method) deal with reduction of discrete-time models, the systems are discretized. The discretization is performed, while warping around a particular frequency $\omega_0$ in the process:

$$s = \lambda \frac{z - 1}{z + 1}, \quad \lambda = \frac{\omega_0}{\tan(\omega_0 T_s/2)}$$

If the Nyquist sampling time is bigger than $T_s$, then non of the dynamics are lost. $\omega_0$ is a tuning parameter for numerical conditioning. For example, if the largest peak in magnitude occurs around a frequency $\omega_1$, then prewarping around this frequency will create a better numerically conditioned problem.

The Hankel model reduction is implemented by the MATLAB™ routine HANKELMR using the procedure (1) afterwards. All the frequency samples matching methods are implemented using a cutting plane algorithm (for more details, see [16]). For the optimization algorithms, initially a uniform on the interval $[0, \pi]$ frequency grid is taken with extra points added around the frequencies that correspond to the peaks in magnitude of the full model.

### Table I

<table>
<thead>
<tr>
<th>Approximation errors in percent in Example 1</th>
<th>Reduction order k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of points in the grid</td>
<td>400</td>
</tr>
<tr>
<td>$\sigma_{\infty}(G)$</td>
<td>0.17%</td>
</tr>
<tr>
<td>HMR</td>
<td>2.07%</td>
</tr>
<tr>
<td>QCO</td>
<td>1.93%</td>
</tr>
<tr>
<td>OHMR</td>
<td>1.93%</td>
</tr>
<tr>
<td>PRD with HMR poles</td>
<td>1.93%</td>
</tr>
<tr>
<td>PRD with QCO poles</td>
<td>1.92%</td>
</tr>
<tr>
<td>PRD with OHMR poles</td>
<td>1.92%</td>
</tr>
<tr>
<td>PRD with poles of $G$</td>
<td>1.93%</td>
</tr>
<tr>
<td>PRD with a structure</td>
<td>2.18%</td>
</tr>
</tbody>
</table>

**Example 1: Deformable Mirror Modeling.** The following model was studied in [17] and obtained by means of the finite element modeling approach that resulted in a system of second-order differential equations:

$$M \ddot{x} + D \dot{x} + K x = Bu$$

$$y = C x$$

where matrices $M$, $K$, $D$ have high dimensions which results in a state-space model with more than 20000 states.

In [17] a heuristic reduction have been performed and the obtained model has 2000 states. In the process the system becomes mass-normalized i.e., $M = I$, and $K = \Lambda^2$, where $\Lambda$ is a diagonal matrix. Damping is chosen as $D = 0.02\Lambda$ and $C = B^T$.

The frequency responses of the full model and its 8-th order approximations are depicted in Fig. 1. The $\mathcal{H}_\infty$ norms of the approximation errors for orders 8 and 16 are also presented in Table I. Hankel model reduction in this examples showed worse performance both in accuracy and computational speed. It is most likely that the accuracy was affected by the high order of the full model. Note also, that in Fig. 1 the 8-th order OHMR approximation (thick black line) follows the phase of the full model (thick gray line) much better than the HMR approximation with the corresponding order (dashed blue line).
In this example identifying the poles of the original model $G$ was not hard, since the matrices $M, K$ and $D$ are diagonal and one needs to solve decoupled quadratic equations in order to compute the eigenvalues.

By PRD with a structure is assumed the algorithm with two zeros fixed at $\omega = \pi$. This was implemented, since any systems with structure as (17) would have such zeros in any discrete-time realization.

Example 2: Controller Reduction. Consider the control system $P(G, K)$, where the plant $G$ is controlled in a robust manner by a controller $K$:

$$P(G, K) = \begin{pmatrix} 1 + KG & -KG \\ 1 & 1 + KG \\ 1 + KG & 1 + KG \end{pmatrix}$$

The 152-nd order controller $K$ was obtained in [18] using Youla parameterization, therefore the controller itself is stable and so is the third order plant $G = b/a$, where $b$ and $a$ are the polynomials in $e^{-j\omega}$. Define the reduced order controller $\hat{K} = p/q$, where $p, q$ are the polynomials of order $k$ in $e^{-j\omega}$, and

$$c(p, q) = p(\omega)a(\omega)$$
$$d(p, q) = q(\omega)a(\omega) + p(\omega)b(\omega)$$

To obtain a stabilizing controller solve:

$$\gamma_c = \min_{\gamma>0, p, q} \gamma \text{ subject to}$$
$$|(P(\omega)d(p, q) - c(p, q))\varphi(\omega)| < \gamma \text{Re}(d(p, q)\varphi(\omega)) \forall \omega$$

The parameterization is convex and it guarantees the close loop stability. If required some of the poles or zeros of the controller may be fixed (in this example one of the poles may be fixed to 1) or constrained to a particular region through the choice of $\varphi$. The obtained $p$ and $q$ yield a closed loop error bound:

$$\|P(G, K) - P(G, \hat{K})\| \leq \gamma_c$$

The major design parameter in this algorithm is a function $\varphi$. In this particular case it was chosen as follows. $P_r = f/e$ is a $k + 3$-rd order approximation of $P$ obtained by balanced truncation. Since the closed loop system is stable, so is the polynomial $e$. The parameter $\varphi$ is chosen as $\varphi = e^{\gamma}/|e|$. The best choice of $\varphi$ for this application is yet to be determined and is a subject to further investigation. For this particular example several methods to define $\varphi$ were used and only a marginally different performance was observed.

The performance of various methods for different order reduction is presented in Fig. 2. The structured Gramian approach from [19] did not provide a solution. For the frequency weighted balanced truncation and QCO algorithm the weights are chosen as follows The input weight is $W_z = (1 + KG)^{-1}$ and the output weight is minimum phase and $W_\omega W_\omega^* = GG^* + 1$. PRD is the algorithm described above and finally PRDI is the same algorithm with one of the poles of the controller fixed to $z = 1$, (i.e., the reduced controller has an integrator). The lower reduction bound was calculated as $\sigma_{k+4}(H)$, where $\sigma_i(H)$ is the $i$-th largest Hankel singular value of $H$, $k$ is the order of the reduced controller.

The third order reduced controller with an integrator is of particular interest, since it may be treated as a PID controller. Using the frequency-weighted methods such an approximation has a poor performance, using the proposed techniques the error is reasonable.

Generally, there is no guarantee that the method will provide a stabilizing controller in definitions of [20], e.g., if the plant has an unstable zero or an unstable pole. This is a topic of the future research.

VII. CONCLUSION AND DISCUSSION

In this paper, an approach to model reduction of linear time invariant systems has been presented. The method requires only the frequency response samples to obtain an approximation and guarantees stability of one in $H_\infty$ sense. The minimization is performed in a Hankel-type norm, therefore the accuracy of the algorithm is expected to be close to the optimal Hankel one. The method may be also applied to reduction of structured systems, e.g. plant-controller structure. The quality of the relaxation is validated on the numerical examples.

A multivariable (MIMO) extension of the proposed method can be done using the techniques similar to [21]. This solution may be restrictive for a large number of inputs. Some improvement was achieved using the bilinear reformulation, however, no systematic semidefinite method was discovered by the author.

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