Schedulability and Optimization Analysis for Non-Preemptive Static Priority Scheduling Based on Task Utilization and Blocking Factors

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Preemptive vs. Non-Preemptive Scheduling

**Preemptive**

- $\tau_1$
- $\tau_2$

**Non-Preemptive**

- $\tau_1$
- $\tau_2$ blocks $\tau_1$

*Deadline Miss*
Preemptive vs. Non-Preemptive Scheduling

\( \tau_1 \)

Preemptive

\( \tau_2 \)

Non-Preemptive

\( \tau_2 \) blocks \( \tau_1 \)

Deadline Miss

v. d. Brüggen, Chen, Huang  (LS 12, TU Dortmund)
Preemptive vs. Non-Preemptive Scheduling

Preemptive Schedule:
- Process $\tau_1$ starts and runs until it is preempted.
- Process $\tau_2$ starts, runs for a period, and blocks $\tau_1$.
- $\tau_1$ resumes, runs for a period, and misses its deadline.

Non-Preemptive Schedule:
- Process $\tau_1$ starts and runs until it is pre-empted by $\tau_2$.
- Process $\tau_2$ blocks $\tau_1$.
- Both processes miss their deadlines due to block.
Notation

\[ \tau_i(C_i, D_i, T_i), \quad U_i = \frac{C_i}{T_i} \]
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Blocking Factor \( \gamma_k \):

\[ \frac{B_k}{C_k} \]

Blocking Time \( B_k \)
Notation

\[ \tau_i(C_i, D_i, T_i), \ U_i = \frac{C_i}{T_i} \]

**Blocking Factor** \( \gamma_k : \)

\[ B_k : \text{Blocking Time} \]

**Task Set Blocking Factor** \( \gamma : \max_{\tau_k \in \tau} \{ \gamma_k \} \)
Interesting Problem: Speedup Factors

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Fixed Priority Scheduling Speedup Factors

[R. Davis, L. George, P. Courbin. Quantifying the sub-optimality of uniprocessor fixed priority non-pre-emptive scheduling. 2010.]

- $\Omega = \ln \left( \frac{1}{\Omega} \right)$ (transcendental equation)
- $\Omega \approx 0.56714$
- $\frac{1}{\Omega} \approx 1.76322$
Total utilization bounds of RM-NP with respect to $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in l_p(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$

provided Andersson and Tovar (2009)
Response Time of $\tau_k$ (Example: $\tau_4$)

$\tau_1 = (2, 8, 8)$

$\tau_2 = (2, 10, 10)$

$\tau_3 = (3, 12, 12)$

$\tau_4 = (3, 23, 35)$

$\tau_b = (2, 99, 99)$

The schedule according to Deadline Monotonic

$\exists t$ with $0 < t \leq D_k - C_k$ and $B_k + \sum_{i=1}^{k-1} \left( \left\lfloor \frac{t}{T_i} \right\rfloor + 1 \right) C_i \leq t$
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$\Rightarrow \exists t$ with $0 < t \leq D_k - C_k$ and $B_k + \sum_{i=1}^{k-1} \left\lfloor \frac{t}{T_i} \right\rfloor C_i \leq t$
Self-Pushing Phenomenon

The worst-case response time of a non-preemptive task occurs in the first job if the task is activated at its critical instant and the following two conditions are both satisfied:

1. the task set is feasible under preemptive scheduling;
2. the relative deadlines are less than or equal to periods.
Preemptive and Non-Preemptive Case Simultaneously

\[ \tau_1 = (2, 8, 8) \]
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The schedule according to Deadline Monotonic

\[ \text{NP: } \exists t \text{ with } 0 < t \leq D_k - C_k \]
\[ \text{and } B_k + \sum_{i=1}^{k-1} \left\lceil \frac{t}{T_i} \right\rceil C_i \leq t \]

\[ \text{P: } \exists t \text{ with } 0 < t \leq D_k \]
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The schedule according to Deadline Monotonic

NP: $\exists t$ with $0 < t \leq D_k - C_k$ and $B_k + \sum_{i=1}^{k-1} \left\lfloor \frac{t}{T_i} \right\rfloor C_i \leq t$

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NP+P: $\exists t$ with $0 < t \leq D_k$ and $B_k + C_k + \sum_{i=1}^{k-1} \left\lfloor \frac{t}{T_i} \right\rfloor C_i \leq t$
Aims of the Paper

\( NP^+P: \exists t \text{ with } 0 < t \leq D_k \text{ and } B_k + C_k + \sum_{i=1}^{k-1} \left\lfloor \frac{t}{T_i} \right\rfloor C_i \leq t \)

- Easy sufficient schedulability test
  - Good Runtime: \( O(k) \)
  - Utilization of higher priority tasks
  - Blocking time
- Hyperbolic form
  - Good Runtime: \( O(n) \)
- Speedup Factor
  - Non-preemptive deadline monotonic
  - Non-preemptive rate monotonic
- Utilization bounds for non-preemptive rate monotonic
  - Depending on blocking factor
  - Blocking factor \( \leq 2 \)
Sufficient Linear Time Schedulability Test for $\tau_k$

$\tau_1 = (2, 8, 8)$

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The schedule according to Deadline Monotonic

$\exists t_j \in \{t_1, t_2, \ldots, t_{k-1}, t_k\}$ and $B_k + C_k + \sum_{i=1}^{k-1} \left\lceil \frac{t_j}{T_i} \right\rceil C_i \leq t_j$
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Splitting the Summation

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\[ \exists t_j \in \{t_1, \ldots, t_k\} \quad \text{and} \quad B_k + C_k + \sum_{i=1}^{k-1} \left\lceil \frac{t_j}{T_i} \right\rceil C_i \leq t_j \]
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Releases before last:
\[ \exists t_j \in \{ t_1, \ldots, t_k \} \text{ and } B_k + C_k + \sum_{i=1}^{k-1} \left\lceil \frac{t_j}{T_i} \right\rceil C_i \leq t_j \]
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\[ \Rightarrow \exists t_j \in \{ t_1, \ldots, t_k \} \text{ and } B_k + C_k + \sum_{i=1}^{k-1} t_i U_i + \sum_{i=1}^{j-1} t_i U_i \leq t_j \]
Test for $\tau_k$ in Hyperbolic Form (Theorem 1)

- Non-preemptive sporadic task system
- Constrained deadlines
- Fixed priority scheduling
- Schedulability of higher priority tasks has already been ensured

**Theorem 1**

A task $\tau_k$ is schedulable if:

$$\left( \frac{B_k + C_k}{D_k} + 1 \right) \prod_{\tau_j \in \text{hp}(\tau_k)} (U_j + 1) \leq 2$$

Runtime: $O(n)$
### Speedup Factor for DM-NP and RM-NP

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Utilization Bound for RM-NP

**Theorem 3**
Suppose that the tasks are indexed such that \( T_i \leq T_{i+1} \). If \( \gamma = \max_{\tau_i \in Ip(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} = \frac{B_k}{C_k} \), then task \( \tau_k \) is schedulable by RM-NP if

\[
U_{sum} \leq \begin{cases} 
\left( \left( \frac{2}{1+\gamma} \right)^{\frac{1}{k}} - \frac{1}{1+\gamma} \right) + (k-1) \left( \left( \frac{2}{1+\gamma} \right)^{\frac{1}{k}} - 1 \right) & \text{if } \gamma \leq 1 \\
\frac{1}{1+\gamma} & \text{if } \gamma > 1
\end{cases}
\]

**Theorem 4**
Suppose that \( \gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in Ip(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\} \). A task set can be feasibly scheduled by RM-NP if

\[
U_{sum} \leq \begin{cases} 
\frac{\gamma}{\gamma+1} + \ln \left( \frac{2}{1+\gamma} \right) & \text{if } \gamma \leq 1 \\
\frac{1}{1+\gamma} & \text{if } \gamma > 1
\end{cases}
\]
Utilization Bound for RM-NP

Total utilization bounds of RM-NP with respect to $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in lp(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$

Comparison: Andersson and Tovar (2009) and Theorem 4
Tighter Schedulability Test

Testing schedulability in two equations:

**NP:** \( \exists t \ 0 < t \leq D_k - C_k \) and \( B_k + \sum_{i=1}^{k-1} \left\lceil \frac{t_j}{T_i} \right\rceil C_i \leq t \)

**P:** \( \exists t \ 0 < t \leq D_k \) and \( C_k + \sum_{i=1}^{k-1} \left\lceil \frac{t_j}{T_i} \right\rceil C_i \leq t \)

**Theorem 6**

A task \( \tau_k \) is schedulable by a fixed priority non-preemptive scheduling algorithm \( A \) if all higher priority tasks are schedulable and the following two conditions hold:

\[
\left( \frac{B_k}{D_k - C_k} + 1 \right) \prod_{\tau_j \in hp(\tau_k)} (U_j + 1) \leq 2 \\
\left( \frac{C_k}{D_k} + 1 \right) \prod_{\tau_j \in hp(\tau_k)} (U_j + 1) \leq 2
\]
Theorem 9

Suppose that $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in lp(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$. A task set can be feasibly scheduled by RM-NP if

$$U_{sum} \leq \begin{cases} \ln(2) \approx 0.693 & \text{if } \gamma \leq \frac{1-\ln(2)}{\ln(2)} \\ \frac{1}{1+\gamma} & \text{if } \gamma > \frac{1-\ln(2)}{\ln(2)} \end{cases}$$
Total utilization bounds of RM-NP with respect to $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in \text{lp}(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$ provided by Andersson and Tovar, Theorem 4 and Theorem 9.
Results

- First schedulability test in hyperbolic form for non-preemptive fixed priority scheduling
- Tighter schedulability test based on two individual equations in hyperbolic form
- Tight speedup factors of $\frac{1}{\Omega} \approx 1.76322$ for Deadline Monotonic (DM) and Rate Monotonic (RM) non-preemptive scheduling in comparison to non-preemptive Earliest Deadline First (EDF)
- Utilization bounds for Rate Monotonic scheduling depending on the blocking factor $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in l_p(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$ if $\gamma < 2$
Results

• First schedulability test in hyperbolic form for non-preemptive fixed priority scheduling
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  if $\gamma < 2$

Thank You!